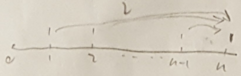
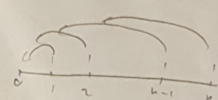
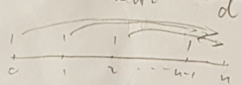
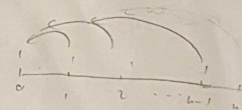
Tutorial 3.

Preliminary:

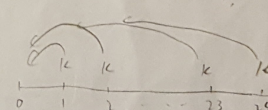
| | Accumulated Value | Present Value |
|---|---|---|
| Annuity-Immediate (end of the period) | $s_{\overline{n}\vert i} = (1+i)^{n-1} + \cdots + (1+i) + 1 = \frac{(1+i)^n - 1}{i}$  | $a_{\overline{n}\vert i} = v + v^2 + \cdots + v^n = \frac{1-v^n}{i}$  |
| Annuity-Due (beginning of the period). | $\ddot{s}_{\overline{n}\vert i} = (1+i)^n + \cdots + (1+i)^2 + (1+i) = (1+i)s_{\overline{n}\vert i} = \frac{(1+i)^n - 1}{d}$  | $\ddot{a}_{\overline{n}\vert i} = 1 + v + \cdots + v^{n-1} = \frac{1-v^n}{1-v} = \frac{1-v^n}{d}$.  |

2.1.1.

Option 1: Accumulate Value $= 50{,}000(1+i)^{24}$.

Option 2: Annuity Payment is $K$, $\quad K a_{\overline{24}\vert 10\%} = 50{,}000 \Rightarrow K = 5564.99$.

$K s_{\overline{24}\vert 5\%} = 50{,}000(1+i)^{24}, \Rightarrow i = 6.9\%$.



$50{,}000 = K a_{\overline{24}\vert 10\%} \Rightarrow K = 5564.99$.

2.2.1.

(a) monthly rate $j = (1+i)^{\frac{1}{12}} - 1$, $\quad i$ is annual rate,

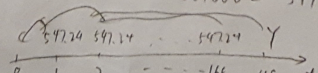
and $\quad 1+i = [1 + \frac{i^{(2)}}{2}]^2$, $\quad$ then $\quad j = ((1+\frac{i^{(2)}}{2})^2)^{\frac{1}{12}} - 1 = (1+\frac{i^{(2)}}{2})^{\frac{1}{6}} - 1 = (1+\frac{10\%}{2})^{\frac{1}{6}} - 1$

$X$ is monthly payment, $\quad X a_{\overline{300}\vert j} = 50{,}000, \Rightarrow X = 447.24$.

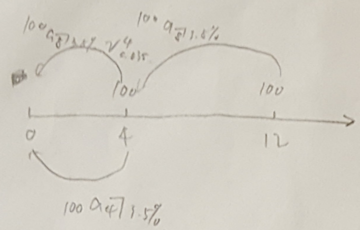
(b) $\quad (X+100) a_{\overline{n}\vert j} = 50{,}000 \Rightarrow a_{\overline{n}\vert j} = 91.37. \Rightarrow n = 168.5$.

$168^{th}$ payment occurs on Dec 31, 2023 by $X + 100 = 547.24$.

The fractional part $\quad 50{,}000 - 547.24 a_{\overline{168}\vert j} = Y v_j^{169} \Rightarrow Y = 290.30$.



2.7.11.




$$1000 = 100\, a_{\overline{4}|0.035} + 100\, a_{\overline{8}|0.035} \cdot v^4_{0.035}$$

$$\Rightarrow a_{\overline{8}|i} = 7.26 \Rightarrow i = 7.708\%.$$

2.1.27.

a) $$\ddot{a}_{\overline{n}|i} = \frac{1-v^n}{d} = \frac{1-v^n}{\frac{i}{1+i}} = (1+i)\cdot\frac{1-v^n}{i} = (1+i)a_{\overline{n}|i} = a_{\overline{n}|i} + i\,a_{\overline{n}|i} = a_{\overline{n}|i} + (1-v^n)$$

$$= v + v^2 + \dots + v^n + 1 - v^n = 1 + v + \dots + v^{n-1} = a_{\overline{n-1}|i}.$$

b) $$\ddot{s}_{\overline{n}|i} = \frac{(1+i)^n-1}{d} = \frac{(1+i)^n-1}{\frac{i}{1+i}} = (1+i)\cdot\frac{(1+i)^n-1}{i} = (1+i)s_{\overline{n}|i}$$

$$= s_{\overline{n}|i} + i\,s_{\overline{n}|i} = s_{\overline{n}|i} + (1+i)^n - 1 = (1+i)^{n-1} + \dots + (1+i) + 1 + (1+i)^n - 1 = s_{\overline{n+1}|i} - 1.$$